

Explaining Zipf's Law via Mental Lexicon

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The Zipf's law is the major regularity of statistical linguistics that served as a prototype for rank-frequency relations and scaling laws in natural sciences. Here we show that the Zipf's law—together with its applicability for a single text and its generalizations to high and low frequencies including hapax legomena—can be derived from assuming that the words are drawn into the text with random probabilities. Their apriori density relates, via the Bayesian statistics, to general features of the mental lexicon of the author who produced the text.

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The Zipf's law states that in a given text the ordered and normalized frequencies $f_1 > f_2 > \dots$ for the occurrence of the word with rank r behave as $f_r \propto r^{-\gamma}$ with $\gamma \approx 1$ [1, 2]. This law applies to texts written in many natural and artificial languages. Its almost universal validity fascinated generations of scholars, but its message is still not well understood: is it just a consequence of simple statistical regularities [3, 4], or it reflects a deeper structure of the text [5]? Many approaches were proposed for deriving the Zipf's law suggesting that it can have different origins. They are divided into two groups.

(1) Certain theories deduce the law from certain general premises of the language [3, 6–10]. The general problem of derivations from this group is that explaining the Zipf's law for the language (and verifying it for a frequency dictionary) does not yet mean to explain the law for a concrete text, where the frequency of the same word varies widely from one text to another and is far from its value in a frequency dictionary [12].

(2) The law can be derived from certain probabilistic models [4, 11–16]. Albeit some of these models assume relevance for realistic text-generating processes [14, 15], their a priori assumed probability structure is intricate, hence the question “why the Zipf's law?” translates into “why a specific probabilistic model?” By far most known probabilistic model is a random text, where words are generated through random combinations of letters and the space symbol seemingly reproducing the $f_r \propto r^{-1}$ shape of the law [3, 4]. But the reproduction is elusive, since the model leads to a huge redundancy—many words have the same frequency and length—absent in normal texts [17].

Our approach for deriving the Zipf's law also uses a probability model. It differs from previous models in several respects. First, it explains the law for a single text together with its limits of validity, i.e. together with the range of ranks where it holds. It also explains the rank-frequency relation for very rare words (hapax legomena)

TABLE I: Parameters of 3 texts: *The Age of Reason* (AR) by T. Paine, 1794 (the major source of British deism). *Thoughts on the Funding System and its Effects* (TF) by P. Ravenstone, 1824 (economics). *Dream Lover* (DL) by J. MacIntyre, 1987 (romance novella). Total number of words N , number of different words n , the lower r_{\min} and the upper r_{\max} ranks of the Zipfian domain, the fitted values of c and γ .

Texts	N	n	r_{\min}	r_{\max}	c	γ
TF	26624	2067	36	371	0.168	1.032
AR	22641	1706	32	339	0.178	1.038
DL	24990	1748	34	230	0.192	1.039

and relates it to the Zipf's law. Second, the a priori structure of our model relates to the mental lexicon [18] of the author who produced the text. Third, the model is not *ad hoc*: it is based on the latent semantic analysis that is used successfully for text modeling.

The validity range of the Zipf's law. Below we present empirical results exemplified on 3 English texts [see Table I] that clarify the validity range of the law, confirm known results, but also make new points that motivate the theoretical model worked out in the sequel.

For each text we extract the ordered frequencies of n different words:

$$\{f_r\}_{r=1}^n, \quad f_1 \geq \dots \geq f_n, \quad \sum_{r=1}^n f_r = 1. \quad (1)$$

To fit $\{f_r\}_{r=1}^n$ to the Zipf's form $\hat{f}_r = cr^{-\gamma}$, we represent the data as $\{y_r(x_r)\}_{r=1}^n$, where $y_r = \ln f_r$ and $x_r = \ln r$, and fit it to the linear form $\{\hat{y}_r = \ln c - \gamma x_r\}_{r=1}^n$. Two unknowns $\ln c$ and γ are obtained from minimizing the sum of squared errors $SS_{\text{err}} = \sum_{r=1}^n (y_r - \hat{y}_r)^2$ [28]. Now $\min_{c, \gamma} [SS_{\text{err}}] = SS_{\text{err}}^*$ and the correlation coefficient R^2 between $\{y_r\}_{r=1}^n$ and $\{\hat{y}_r\}_{r=1}^n$ [20, 28] measure the fitting quality: $SS_{\text{err}}^* \rightarrow 0$ and $R^2 \rightarrow 1$ mean good fitting. We minimize SS_{err} over c and γ for $r_{\min} \leq r \leq r_{\max}$ and find the maximal value of $r_{\max} - r_{\min}$ for which SS_{err}^*

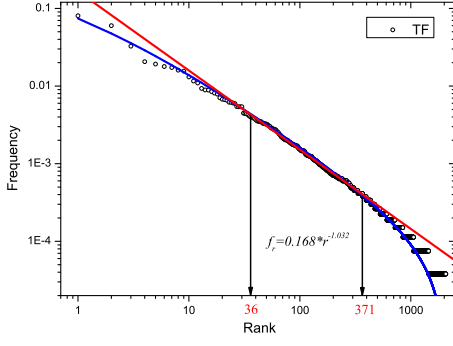


FIG. 1: (Color online) Frequency vs. rank for the text TF; see Table I. Red line: the Zipf curve $f_r = 0.168r^{-1.032}$. Arrows indicate on the validity range of the Zipf's law. Blue line: the solution of (9, 10) for $c = 0.168$ and $n = 2067$. It coincides with the generalized Zipf law (14) for $r > r_{\min} = 36$. The step-wise behavior of f_r for $r > r_{\max}$ refers to hapax legomena.

and $1 - R^2$ are smaller than, respectively, 0.05 and 0.005. This value of $r_{\max} - r_{\min}$ also determines the final fitted values of c and γ ; see Table I and [28].

1. For each text there is a specific (Zipfian) range of ranks $r \in [r_{\min}, r_{\max}]$, where the Zipf's law holds with $\gamma \approx 1$ and $c < 0.2$ [1, 2]; see Table I and Fig. 1.

2. Even if the same word enters into different texts it typically has quite different frequencies there [12], e.g. among 83 common words in the Zipfian ranges of AR and DL [see Table I], only 12 words have approximately equal ranks and frequencies.

3. The pre-Zipfian $1 \leq r < r_{\min}$ range contains mainly function words. They serve for establishing grammatical constructions (e.g., *the, a, such, this, that, where, were*). But the majority of words in the Zipfian range do have a narrow meaning (content words). A subset of those content words has a meaning that is specific for the text and can serve as its keywords [21]. Below [in 15] we explain why the key-words appear in the Zipfian domain.

4. The absolute majority of different words with ranks in $[r_{\min}, r_{\max}]$ have different frequencies. Only for $r \simeq r_{\max}$ the number of different words having the same frequency is $\simeq 10$. For $r > r_{\max}$ we meet the *hapax legomena*: words occurring only few times in the text ($f_r N = 1, 2, \dots$ is a small integer), and many words having the same frequency f_r [2]. The effect is not described by a smooth rank-frequency relation, including the Zipf's law.

5. The minimal frequency of the Zipfian domain holds $f_{r_{\max}} > c/n$. We checked that this is valid not only for separate texts but also for the frequency dictionaries of English and Irish. For our texts a stronger relation holds $f_{r_{\max}} \gtrsim \frac{1}{n}$. Hence $f_{r_{\max}} N \gtrsim \frac{N}{n} \gg 1$; see Table I.

Introduction to the model. A model for the Zipf's law is supposed to satisfy the following features.

(I) Apply to separate texts, i.e. explain how different

texts can satisfy the same form of the rank-frequency relation despite the fact that the same words do *not* occur with same frequencies in the different texts; see 2.

(II) Derive the law together with its extensions for all frequencies, limits of validity and hapax legomena effect.

(III) Relate the law to formation of a text.

Two sources of the model are the latent semantic analysis [22], and the idea of applying ordered statistics for rank-frequency relations [8, 24, 25].

Our model makes four (A – D) assumptions.

A. The *bag-of-words picture* focusses on the frequency of the words that occur in a text and neglects their mutual disposition (i.e. syntactic structure) [23]. Given n different words $\{w_k\}_{k=1}^n$, the joint probability for w_k to occur $\nu_k \geq 0$ times in a text T is multinomial

$$\pi[\nu|\theta] = \frac{N! \theta_1^{\nu_1} \dots \theta_n^{\nu_n}}{\nu_1! \dots \nu_n!}, \quad \nu = \{\nu_k\}_{k=1}^n, \quad \theta = \{\theta_k\}_{k=1}^n, \quad (2)$$

where $N = \sum_{k=1}^n \nu_k$ is the length of the text, ν_k is the number of occurrences of w_k , and θ_k is the probability of w_k . The picture is well-known in computational linguistics [23]. But for our purposes it is incomplete, because it implies that each word has the same probability for different texts [recall (I)].

B. To improve this point we make θ a random vector [23] with a text-dependent density $P(\theta|T)$. The simplest assumption is that (T, θ, ν) form a Markov chain: the text T influences the observed ν only via θ . Then the probability $p(\nu|T)$ of ν in a given text T reads

$$p(\nu|T) = \int d\theta \pi[\nu|\theta] P(\theta|T). \quad (3)$$

This form of $p(\nu|T)$ is basic for probabilistic latent semantic analysis [22], a successful method of computational linguistics. There the density $P(\theta|T)$ of latent variables θ is determined from the data fitting. But we shall deduce $P(\theta|T)$ theoretically.

C. $P(\theta|T)$ is generated from a density $P(\theta)$ via conditioning on the ordering of $\mathbf{w} = \{w_k\}_{k=1}^n$ in T :

$$P(\theta|T) = P(\theta) \chi_T(\theta, \mathbf{w}) / \int d\theta' P(\theta') \chi_T(\theta', \mathbf{w}). \quad (4)$$

If different words of T are ordered as (w_1, \dots, w_n) with respect to the decreasing frequency of their occurrence in T (i.e. w_1 is more frequent than w_2), then $\chi_T(\theta, \mathbf{w}) = 1$ if $\theta_1 \geq \dots \geq \theta_n$, and $\chi_T(\theta, \mathbf{w}) = 0$ otherwise.

As substantiated below, $P(\theta)$ refers to the mental lexicon of the author prior to generating a concrete text.

D. For simplicity, we assume that the probabilities θ_k are distributed identically and the dependence among them is due to $\sum_{k=1}^n \theta_k = 1$ only:

$$P(\theta) \propto u(\theta_1) \dots u(\theta_n) \delta(\sum_{k=1}^n \theta_k - 1), \quad (5)$$

where $\delta(x)$ is the delta function and the normalization ensuring $\int_0^\infty \prod_{k=1}^n d\theta_k P(\theta) = 1$ is omitted.

Solution of the model and the Zipf's law. The conditional probability $p_r(\nu|T)$ for the r 'th most frequent word w_r to occur ν times in the text T reads from (2, 3)

$$p_r(\nu|T) = \frac{N!}{\nu!(N-\nu)!} \int_0^1 d\theta \theta^\nu (1-\theta)^{N-\nu} P_r(\theta|T), \quad (6)$$

$$P_r(t|T) = \int d\theta P(\theta|T) \delta(t - \theta_r), \quad (7)$$

where $P_r(t|T)$ is the marginal density for the probability t of w_r . For $n \gg 1$, we deduce from (4, 5) that $P_r(t|T)$ follows the law of large numbers [28]. It is Gaussian,

$$P_r(t|T) \propto \exp\left[-\frac{n^3}{2\sigma_r^2}(t - \phi_r)^2\right], \quad (8)$$

where $\sigma_r = \mathcal{O}(1)$ [for $\phi_r = o(1)$], and the mean ϕ_r is found from two equations for two unknowns μ and ϕ_r :

$$r/n = \int_{\phi_r}^{\infty} d\theta u(\theta) e^{-\mu\theta n} / \int_0^{\infty} d\theta u(\theta) e^{-\mu\theta n}, \quad (9)$$

$$\int_0^{\infty} d\theta \theta u(\theta) e^{-\mu\theta n} = \frac{1}{n} \int_0^{\infty} d\theta u(\theta) e^{-\mu\theta n}. \quad (10)$$

Eq. (8) holds for $P_r(t|T)$ whenever its standard deviation $\sigma_r n^{-3/2}$ is much smaller than the mean ϕ_r ; as checked below, this happens already for $r > 10$.

6. The meaning of (9, 10) is explained via the marginal density $P(\theta_1) = \int_0^{\infty} \prod_{k=2}^n d\theta_k P(\theta) \propto u(\theta_1) e^{-\mu\theta_1 n}$ found from (5) [28]. Eq. (10) ensures that $\int_0^{\infty} d\theta \theta P(\theta) = \frac{1}{n}$. This relation follows from $\sum_{k=1}^n \theta_k = 1$ and it determines μ , an analogue of the chemical potential in statistical physics [28]. The interpretation of (9) is that it equates the relative rank r/n to the (unconditional) probability $\int_{\phi_r}^{\infty} d\theta P(\theta)$ of $\theta \geq \phi_r$.

Let us study implications of (6–10) for the Zipf's law.

7. In (6), $P_r(\theta|T)$ is much more narrow peaked than $\theta^\nu(1-\theta)^{N-\nu}$, since $n^3 \gg N \gg 1$ [see Table I]. Hence in this limit we approximate $P_r(\theta|T)$ by delta-function $\delta(\theta - \phi_r)$ [see (8)]:

$$p_r(\nu|T) = \frac{N!}{\nu!(N-\nu)!} \phi_r^\nu (1-\phi_r)^{N-\nu}. \quad (11)$$

Eq. (11) is the main outcome of the model; it shows that the conditional probability $p_r(\nu|T)$ for the occurrence number ν of the word w_r has the same form (11) for different text (see **I**). In (11), ϕ_r is the effective probability of the word w_r . If $N\phi_r \gg 1$, $p_r(\nu|T)$ is peaked at $\nu = N\phi_r$: the frequency of a word that appears many times equals its probability. Each word of the Zipfian domain occurs at least $\nu \sim N/n \gg 1$ times; see **5**. For such words we approximate $f_r \equiv \nu/N \simeq \phi_r$.

8. Now we postulate in (5)

$$u(f) = (n^{-1}c + f)^{-2}, \quad (12)$$

where c is related below to the prefactor of the Zipf's law. Eq. (12) is explained in **13–15** below.

TABLE II: Description of the hapax legomena for the text TF; see Table I and (15). The maximal relative error $\frac{\hat{r}_k - r_k}{r_k} = 0.0357$ is reached for $k = 6$.

r/k	1	2	3	4	5	6	7	8	9	10
r_k	1446	1061	848	722	611	529	474	437	398	370
\hat{r}_k	1414	1074	866	726	624	547	488	440	400	368

9. For $c \lesssim 0.2$, $c\mu$ determined from (10, 12) is small and is found from integration by parts:

$$\mu \simeq c^{-1} e^{-\gamma_E - \frac{1+c}{c}}, \quad (13)$$

where $\gamma_E = 0.55117$ is the Euler's constant. One solves (9) for $c\mu \rightarrow 0$: $\frac{r}{n} = ce^{-n\phi_r\mu}/(c + n\phi_r)$. For $r > r_{\min}$, $\phi_r n\mu = f_r n\mu < 0.04 \ll 1$; see (13) and Table I. We get

$$f_r = c(r^{-1} - n^{-1}). \quad (14)$$

This is the Zipf's law generalized by the factor n^{-1} at high ranks r . This cut-off factor ensures faster [than r^{-1}] decay of f_r for large r . In literature a cut-off factor similar to $\frac{1}{n}$ is introduced due to additional mechanisms (hence new parameters); see [14]. In our situation the power-law and cut-off come from the same mechanism.

Fig. 1 shows that (14) reproduces well the empirical behavior of f_r for $r > r_{\min}$. Our derivation shows that c is the prefactor of the Zipf's law, and that our assumption on $c < 0.2$ above (13) agrees with observations; see Table I. For $c \gg 0.2$, (9, 10) do not predict the Zipf's law (14).

10. For given prefactor c and the number of different words n , (9–12) predict the Zipfian range $[r_{\min}, r_{\max}]$ in agreement with empirical results; see Fig. 1.

11. For $r < r_{\min}$, it is not anymore true that $f_r n\mu \ll 1$. So the fuller expression (9) is to be used. It reproduces qualitatively the empiric behavior of f_r ; see Fig. 1.

12. According to (11), the probability ϕ_r is small for $r \gg r_{\max}$ and hence the occurrence number $\nu \equiv f_r N$ of a words w_r is a small integer (e.g. 1 or 2) that cannot be approximated by a continuous function of r ; see (12) and Fig. 1. To describe this hapax legomena range, define r_k as the rank, when $\nu \equiv f_r N$ jumps from integer k to $k+1$. Since ϕ_r reproduces well the trend of f_r even for $r > r_{\max}$, see Fig. 1, r_k can be theoretically predicted from (14) by equating its left-hand-side to k/N :

$$\hat{r}_k = \left[\frac{k}{Nc} + \frac{1}{n}\right]^{-1}, \quad k = 0, 1, 2, \dots \quad (15)$$

Eq. (15) is exact for $k = 0$, and agrees with r_k for $k \geq 1$; see Table II. Hence it describes the hapax legomena phenomenon (many words have the same small frequency) [26].

Preliminary summary. Thus **9–12** achieved the promises (**I**) and (**II**) of our program: though different

texts can have different frequencies for same words, the frequencies of words in a given text follow the Zipf's law with the correct prefactor $c \lesssim 0.2$. Without additional fitting parameters and new mechanisms we recovered the corrected form of this law applicable for large and small frequencies [see 11, 12]. But why we would select (12), if we would not know that it reproduces the Zipf's law? Answering this question will fulfil (III).

Mental lexicon and the apriori density. Here we explain the choice (5, 12) for the apriori probability density for the probabilities $\theta = (\theta_1, \dots, \theta_n)$ of different words (w_1, \dots, w_n) . To avoid the awkward term “probability for probability” we shall call $P(\theta)$ likelihood. We focus on the marginal likelihood [see 6 and (12)]:

$$P(\theta) = (n^{-1}c + \theta)^{-2} e^{-\mu n \theta}, \quad (16)$$

since $P(\theta)$ determines the rank-frequency relation (9). For a more detailed discussion of the items below see [28].

13. The basic reason for the words to have random (variable) probabilities is that the text-producing author should be able to compose different texts, where the same word can have very different frequencies [see I]. Hence $P(\theta)$ relates to the prior knowledge (or lexicon) of the author on words. This concept of *mental lexicon* is an established one in psycholinguistics [18, 19].

14. Once each word w_k has to have a variable probability θ_k , there should be a way for the author to increase it, e.g. when the authors decides that w_k should become a keyword of the text. The ensuing relation between the probability vectors θ' (new) and θ (old) should be a group, since the author should be able to come back from θ' to θ when revising the text. Under certain natural conditions, the only such group with parameters τ_k is [27]:

$$\theta'_k = \tau_k \theta_k \left[\sum_{l=1}^n \tau_l \theta_l \right]^{-1}, \quad \tau_k > 0, \quad k = 1, \dots, n, \quad (17)$$

Eq. (17) is a generalized Bayes formula [27, 28]. It is used in the Bayesian statistics for motivating the choice of priors [27], a task related to ours.

If the author wants to increase τ_1 times the probability of the word w_1 , then in (17) $\tau_1 > 1$ and $\tau_{k \geq 2} = 1$:

$$\theta'_1 = \frac{\tau_1 \theta_1}{1 + (\tau_1 - 1) \theta_1}, \quad \theta'_l = \frac{\theta_l}{1 + (\tau_1 - 1) \theta_1}, \quad \text{for } l \geq 2. \quad (18)$$

The inverse of (18) is found by interchanging θ'_k with θ_k and τ_1 with τ_1^{-1} . For the Zipf's law the relevant probabilities are small, $\theta'_1 < \mathcal{O}(1/n)$; see 9 and Fig. 1. Then $1 + (\tau_1^{-1} - 1) \theta'_1 \approx 1$ and (18) becomes the scaling transformation of one variable: $\theta'_1 = \tau_1 \theta_1$, $\theta'_l = \theta_l$, $l \geq 2$. The new likelihood reads from (18, 16)

$$P'(\theta'_1) = \frac{1}{\tau_1} P\left(\frac{\theta'_1}{\tau_1}\right) = \frac{1}{\tau_1} \left(\frac{c}{n} + \frac{\theta'_1}{\tau_1}\right)^{-2}. \quad (19)$$

Other densities do not change $P'(\theta'_l) = P(\theta'_l)$ for $l \geq 2$.

15. Once $P(\theta)$ describes the mental lexicon, and (17) is an operation by which the text is written, we suppose that the features of $P(\theta)$ can be explained by checking its response to (17). For the ratio of the new to the old likelihood of the probability θ'_1 we get from (19)

$$P'(\theta'_1)/P(\theta'_1) = \tau_1 > 1 \quad \text{for } \theta'_1 \gg c\tau_1/n, \quad (20)$$

$$= \tau_1^{-1} < 1 \quad \text{for } \theta'_1 \ll c\tau_1/n. \quad (21)$$

The meaning of (20, 21) is that once the author decides to increase the probability of the word w_1 by τ_1 times, this word will be τ_1 times more likely produced with the higher probabilities, and τ_1 times less likely with smaller probabilities; see (21). This is the mechanism that ensures the appearance of the keywords in the Zipfian range. It is unique to the form (16) of the marginal likelihood, which by itself is due to the form (12) of $u(\theta)$.

If $P(\theta)$ is assumed to reflect the organization of the mental lexicon, then according to (20, 21) this organization is efficient, because the decision on increasing the probability of w_1 translates to increasing the likelihood of larger values of the probability. The organization is also stable, since the likelihood at large probabilities increases right at the amount the author planned, not more.

Conclusion. We answer the first question asked in the introduction: the Zipf's law—together with the limits of its validity, its generalization to high and low frequencies and hapax legomena—relates to the stable and efficient organization of the mental lexicon of the text-producing author. Practically, our derivation of the Zipf's law will motivate the usage of prior (12) in the schemes of latent semantic analysis. We expect these schemes to be more efficient for real texts, if the prior structure of the model conforms the Zipf's law. The proposed methods can find applications for studying rank-frequency relations and power laws in other fields.

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Supplementary Material

In this supplementary material to the main text we review the linear fitting method, derive and clarify Eqs.(8-10) from the section ***Solution of the model and the Zipf's law*** of the main text, derive the expression for the marginal probability [Eq. (16) and point 6 of the main text], and discuss in more detail the content of section ***Mental lexicon and the apriori density***. These tasks are carried out in, respectively, sections I, II, III and IV below.

I. LINEAR FITTING

Here we recall the main ideas of the linear fitting method that is employed in the section ***The validity range of the Zipf's law*** of the main text.

Table I of the main text presents 3 texts we studied (we worked out more texts that consistently show the same applicability pattern of the Zipf's law). For each text we extract the ordered frequencies of different words [the number of different words is n ; the overall number of words in a text is N]:

$$\{f_r\}_{r=1}^n, \quad f_1 \geq \dots \geq f_n, \quad \sum_{r=1}^n f_r = 1. \quad (22)$$

We should now see whether the data $\{f_r\}_{r=1}^n$ fits to a power law: $\hat{f}_r = cr^{-\gamma}$. We represent the data as

$$\{y_r(x_r)\}_{r=1}^n, \quad y_r = \ln f_r, \quad x_r = \ln r, \quad (23)$$

and fit it to the linear form $\{\hat{y}_r = \ln c - \gamma x_r\}_{r=1}^n$. Two unknowns $\ln c$ and γ are obtained from minimizing the sum of squared errors:

$$SS_{\text{err}} = \sum_{r=1}^n (y_r - \hat{y}_r)^2. \quad (24)$$

It is known since Gauss that this minimization produces

$$-\gamma^* = \frac{\sum_{k=1}^n (x_k - \bar{x})(y_k - \bar{y})}{\sum_{k=1}^n (x_k - \bar{x})^2}, \quad \ln c^* = \bar{y} + \gamma^* \bar{x}, \quad (25)$$

where we defined

$$\bar{y} \equiv \frac{1}{n} \sum_{k=1}^n y_k, \quad \bar{x} \equiv \frac{1}{n} \sum_{k=1}^n x_k. \quad (26)$$

As a measure of fitting quality one can take:

$$\min_{c, \gamma} [SS_{\text{err}}(c, \gamma)] = SS_{\text{err}}(c^*, \gamma^*) \equiv SS_{\text{err}}^*. \quad (27)$$

This is however not the only relevant quality measure. Another (more global) aspect of this quality is the coefficient of correlation between $\{y_r\}_{r=1}^n$ and $\{\hat{y}_r\}_{r=1}^n$ [29]:

$$R^2 = \frac{[\sum_{k=1}^n (y_k - \bar{y})(\hat{y}_k^* - \bar{\hat{y}}^*)]^2}{\sum_{k=1}^n (y_k - \bar{y})^2 \sum_{k=1}^n (\hat{y}_k^* - \bar{\hat{y}}^*)^2}, \quad (28)$$

where

$$\hat{y}^* = \{\hat{y}_r^* = \ln c^* - \gamma^* x_r\}_{r=1}^n, \quad \bar{\hat{y}}^* \equiv \frac{1}{n} \sum_{k=1}^n \hat{y}_k^*. \quad (29)$$

For the linear fitting (25) the squared correlation coefficient is equal to the coefficient of determination,

$$R^2 = \sum_{k=1}^n (\hat{y}_k^* - \bar{y})^2 / \sum_{k=1}^n (y_k - \bar{y})^2, \quad (30)$$

the amount of variation in the data explained by the fitting [29]. Hence $SS_{\text{err}}^* \rightarrow 0$ and $R^2 \rightarrow 1$ mean good fitting. We minimize SS_{err} over c and γ for $r_{\min} \leq r \leq r_{\max}$ and find the maximal value of $r_{\max} - r_{\min}$ for which SS_{err}^* and $1 - R^2$ are smaller than, respectively, 0.05 and 0.005. This value of $r_{\max} - r_{\min}$ also determines the final fitted values c^* and γ^* of c and γ , respectively; see Tables I, II and Fig. 1. Thus c^* and γ^* are found simultaneously with the validity range $[r_{\min}, r_{\max}]$ of the law. Whenever there is no risk of confusion, we for simplicity refer to c^* and γ^* as c and γ , respectively.

II. DERIVATION OF EQS. (8-10) OF THE MAIN TEXT.

In (7) of the main text we defined $P_r(t|T)$: the marginal density for the probability t of the word w_r . Using (4,5) of the main text, we rewrite (7) of the main text as

$$P_r(t|T) \propto \int_0^\infty d\theta_1 \int_0^{\theta_1} d\theta_2 \int_0^{\theta_2} d\theta_3 \dots \int_0^{\theta_{n-1}} d\theta_n \times P(\theta_1, \dots, \theta_n) \delta(t - \theta_r), \quad (31)$$

where

$$P(\boldsymbol{\theta}) \propto u(\theta_1) \dots u(\theta_n) \delta(\sum_{k=1}^n \theta_k - 1), \quad (32)$$

as given by (7) of the main text. Recall that $\boldsymbol{\theta} = (\theta_1, \dots, \theta_n)$.

In (32) we employ the Fourier representation of the delta-function,

$$\delta(\sum_{k=1}^n \theta_k - 1) = \int_{-i\infty}^{i\infty} \frac{dz}{2\pi i} e^{z - z \sum_{k=1}^n \theta_k}, \quad (33)$$

put (32) into (31) and then apply integration by parts. The result reads

$$P_r(t|T) \propto u(t) \int_{-i\infty}^{i\infty} \frac{dz e^z}{2\pi i} \chi_0^{n-r}(t, z) \chi_1^{r-1}(t, z) e^{-tz}, \quad (34)$$

where

$$\chi_0(t, z) \equiv \int_0^t dy e^{-zy} u(y), \quad \chi_1(t, z) \equiv \int_t^\infty dy e^{-zy} u(y).$$

The integral in (34) will be worked out via the saddle point method. But before that we need to fix the scales of the involved quantities. To this end, make the following changes of variables

$$\tilde{z} = z/n, \quad \tilde{t} = tn, \quad \tilde{y} = yn, \quad \tilde{r} = r/n. \quad (35)$$

Then $P_r(t|T)$ reads from (34)

$$P_r(t|T) \propto u(t) \int_{-i\infty}^{i\infty} \frac{d\tilde{z}}{2\pi i} e^{n\varphi(\tilde{t}, \tilde{z}) - \tilde{t}\tilde{z}}, \quad (36)$$

$$\begin{aligned} \varphi(\tilde{t}, \tilde{z}) &= \tilde{z} + (1 - \tilde{r}) \ln \int_0^{\tilde{t}} \frac{dy e^{-\tilde{z}y}}{(c + y)^2} \\ &+ (\tilde{r} - \frac{1}{n}) \ln \int_{\tilde{t}}^\infty \frac{dy e^{-\tilde{z}y}}{(c + y)^2}, \end{aligned} \quad (37)$$

where in (37) we already used $u(t) = (n^{-1}c + t)^{-2}$; see (12) of the main text.

If $n \gg 1$ and $0 < \tilde{r} < 1$ is a finite number (neither close to one, nor to zero), the behavior of $\rho_r(t)$ in various averages, e.g. $\int dt t \rho_r(t)$, is determined by the values of $\tilde{z} = \tilde{z}_s$ and $\tilde{t} = \tilde{t}_s$ that maximize $\phi(\tilde{t}, \tilde{z})$. They are found from saddle-point equations

$$\partial_{\tilde{t}} \phi(\tilde{t}_s, \tilde{z}_s) = \partial_{\tilde{z}} \phi(\tilde{t}_s, \tilde{z}_s) = 0. \quad (38)$$

After reworking the two equations (38) we get Eqs. (9,10) of the main text.

Due to (35), \tilde{z}_s (that is real and positive) and \tilde{t}_s stay finite for $n \gg 1$. Hence the integration line over \tilde{z} in (36) is shifted to pass through \tilde{z}_s (the saddle-point method). Now $\phi(\tilde{t}, \tilde{z})$ is expanded around $\tilde{z} = \tilde{z}_s$ and $\tilde{t} = \tilde{t}_s$ [first-order terms nullify due to (38)]:

$$\phi(\tilde{t}, \tilde{z}) = \phi(\tilde{t}_s, \tilde{z}_s) + \frac{1}{2} \partial_{\tilde{t}\tilde{t}} \phi(\tilde{t}_s, \tilde{z}_s) (\tilde{t} - \tilde{t}_s)^2 \quad (39)$$

$$+ \frac{1}{2} \partial_{\tilde{z}\tilde{z}} \phi(\tilde{t}_s, \tilde{z}_s) (\tilde{z} - \tilde{z}_s)^2 \quad (40)$$

$$+ \partial_{\tilde{t}\tilde{z}} \phi(\tilde{t}_s, \tilde{z}_s) (\tilde{t} - \tilde{t}_s) (\tilde{z} - \tilde{z}_s) + \dots \quad (41)$$

Now only these terms can be retained in the integral over \tilde{z} . Since this integral goes over the imaginary axis, while \tilde{z}_s is real, the integration contour is to be shifted to pass through \tilde{z}_s . For the convergence of the resulting Gaussian integral we need $\frac{1}{2} \partial_{\tilde{z}\tilde{z}} \phi(\tilde{t}_s, \tilde{z}_s) > 0$. Taking this Gaussian integral leads us to [up to factors that either constant or irrelevant for $n \gg 1$]

$$P_r(t|T) \propto e^{-\frac{n}{2\sigma^2} (\tilde{t} - \tilde{t}_s)^2} = e^{-\frac{n^3}{2\sigma^2} (t - \frac{\tilde{t}_s}{n})^2}, \quad (42)$$

$$\frac{1}{\sigma^2} = \frac{[\partial_{\tilde{t}\tilde{z}} \phi(\tilde{t}_s, \tilde{z}_s)]^2}{\partial_{\tilde{z}\tilde{z}} \phi(\tilde{t}_s, \tilde{z}_s)} - \partial_{\tilde{t}\tilde{t}} \phi(\tilde{t}_s, \tilde{z}_s). \quad (43)$$

Hence $P_r(t|T)$ is approximately Gaussian, with the standard deviation $\mathcal{O}(n^{-3/2})$ much smaller than the average for $\tilde{t}_s = \mathcal{O}(1)$.

In working out (43), we shall employ the fact that in (37) $\tilde{z}_s = \mu$ is a small parameter; see (13) of the main text. This produces [up to smaller corrections]

$$\sigma = (c + \tilde{t}_s) \sqrt{\tilde{t}_s}. \quad (44)$$

Eq. (42) derives (8) of the main text, while (44) accounts for the estimate of σ that was presented after (8) of the main text.

III. DERIVATION OF THE MARGINAL PROBABILITY (EQ. (16) AND POINT 6 OF THE MAIN TEXT).

The marginal probability $P(t)$ is defined from (32) as

$$P(t) = \int d\boldsymbol{\theta} P(\boldsymbol{\theta}) \delta(t - \theta_r). \quad (45)$$

using (32, 33) we obtain from (45)

$$P(t) \propto u(t) \int_{-i\infty}^{i\infty} \frac{d\tilde{z}}{2\pi i} e^{n\phi(t, \tilde{z}) - \tilde{t}\tilde{z}}, \quad (46)$$

$$\phi(t, \tilde{z}) = (1 - t)\tilde{z} + \ln \int_0^\infty dy e^{-\tilde{z}y} (c + y)^{-2}. \quad (47)$$

We use the saddle-point method for (46). This produces the same saddle-point equation (38) for \tilde{z}_s ,

$$1 = \frac{\int_0^\infty dy e^{-\tilde{z}_s y} (c + y)^{-2}}{\int_0^\infty dy y e^{-\tilde{z}_s y} (c + y)^{-2}}, \quad (48)$$

provided that we note the dominant range $t \propto 1/n \ll 1$ of t . Thus

$$P(\theta) \propto u(\theta) e^{-n\theta \tilde{z}_s}. \quad (49)$$

This validates Eq. (16) of the main text, as well as its point 6.

Likewise, one can show that the marginal density $P(\theta_1, \dots, \theta_m)$ factorizes provided that $m \ll n$:

$$P(\theta_1, \dots, \theta_m) \propto u(\theta_1) e^{-\mu \theta_1 n} \dots u(\theta_m) e^{-\mu \theta_m n}. \quad (50)$$

Eq. (50) can be established more heuristically via the exact relation $\overline{[\sum_{k=1}^n \theta_k]^2} = 1$, where \overline{f} means averaging over $P(\theta_1, \dots, \theta_n)$. This relation predicts, together with $\theta_k = \frac{1}{n}$, that $\theta_i \theta_j - \theta_i \theta_j = \mathcal{O}(n^{-3})$, hence approximate factorization.

Using (49) with $u(\theta) = (\frac{c}{n} + \theta)^{-2}$ we note that the standard deviation $\langle (\theta - \langle \theta \rangle)^2 \rangle = \frac{1}{n} \sqrt{\frac{c}{\tilde{z}_s} - 1} \simeq \frac{1}{n} \sqrt{\frac{c}{\tilde{z}_s}}$ is larger than the average $\langle \theta \rangle = \int d\theta \theta P(\theta) = \frac{1}{n}$, since $c/\tilde{z}_s \gg 1$.

IV. MENTAL LEXICON AND APRIORI DENSITY

This is an expanded version of the corresponding section of the main text. We explain the choice

$$P(\theta) \propto u(\theta_1) \dots u(\theta_n) \delta(\sum_{k=1}^n \theta_k - 1), \quad (51)$$

$$u(\theta) = (cn^{-1} + \theta)^{-2}, \quad (52)$$

for the apriori probability density for the probabilities $\theta = (\theta_1, \dots, \theta_n)$ of different words w_1, \dots, w_n . To avoid the awkward term “probability for probability” we shall call $P(\theta)$ likelihood.

Recall that the marginal likelihood deduced from (51) reads

$$P(\theta) = (n^{-1}c + \theta)^{-2} e^{-\mu n \theta}, \quad (53)$$

where μ is determined by (12,16) of the main text.

We shall explain the choice (52) via the features of the marginal likelihood (53), because it eventually determines the rank-frequency relation leading to the Zipf’s law.

The numbering of the items 13-15 below coincides that in the section *Mental lexicon and the apriori density* of the main text. The items 13.2, 13.3, 13.4, 16 and 17 below are added additionally, they are absent in the main text.

13.1 Recall that the basic reason for the words to have random (not fixed) probabilities is that the text-producing author should be able to compose different texts, where the same word can have different frequencies. Hence the likelihood $P(\theta)$ of random probabilities

relates to the prior knowledge (or lexicon) of the text-generating author on the words. This concept of *mental lexicon*—the store of words in the long-time memory so that the words are employed on-line for expressing thoughts via phrases and sentences—is well-established in psycholinguistics [30]. Though there is no a unique theory of mental lexicon—there is only a diverse set of competing models [30]—some of its basic features are well-established experimentally and are employed below for explaining the choice (51, 52).

13.2 We assume that during the conceptual planning of the text, i.e. when deciding on its topic, style and potential audience, the author already chooses (at least approximately) two structural parameters: the number n of different words to appear there and the constant c . This is why the marginal likelihood (53) depends on the parameters c and n . We recall that c (along with n) is a structural parameter of the text, because according to the point 5 of the main text, c/n separates the Zipfian (keywords dominated) range from the hapax legomena range (rare words).

13.3 Note that different words have the same marginal likelihood (53). Put differently, the likelihood $P(\theta)$ is symmetric with respect to interchanging the words w_1, \dots, w_n . This feature relates to an experimental fact that words are stored in the mental lexicon in the same way [34]. The difference between them—e.g. whether the word is more familiar to the author, and/or used by him more frequently—can be relevant during the (later) phonologization stage of speech/text production [34]; in this context see also the item 17 below.

Naturally, the above symmetry holds for the apriori likelihood. The posterior likelihood $P(\theta|T)$ (see (6) of the main text), the one that is conditioned over the written text, does not and should not have such a symmetry.

13.4 Note that the marginal likelihood (53) concentrates at small probabilities $\theta \simeq c/n$. The concentration holds locally—since $P(\theta)$ is peaked at $\theta = 0$ and is approximately constant for $\theta \ll c/n$ —and also globally, i.e. on the level of the full probability:

$$\Pr[\theta < a] = \int_0^a d\theta P(\theta) = \int_a^1 d\theta P(\theta) = \Pr[\theta > a], \quad (54)$$

for $a \simeq c/n$.

If a is sufficiently larger (smaller) than c/n , the left-hand-side of (54) is larger (smaller) than its right-hand-side.

The local and global concentrations are different from each other. For example, consider $P(\theta) \propto \theta^{-1/2} e^{-\mu \theta n}$. It displays a local concentration around $\theta \simeq 0$, but (54) (global concentration) predicts $a \simeq \frac{1}{\mu n}$.

Hence according to (53), apriori (i.e. before the text is written) all the (content) words have small probabilities. This is explained as follows. Since the majority of words in the mental lexicon are potential keywords of some texts, apriori (i.e. before the text is written) they have small probabilities. Indeed, the defining (and operationally used) feature of a keyword is that its frequency in a given text is much larger than its frequency in a large mixture of different texts [33]. Thus the apriori likelihood of the probability should be concentrated at small probabilities $\theta \simeq c/n$.

14. Once each word w_k has to have a variable (random) probability θ_k , there should be a way for the author to

change (increase or decrease) this probability, e.g. when the author decides that the word w_k is to become the keyword of the text. The ensuing relation between the probability vectors θ' (new) and θ (old) should be a group, since the author should be able to come back from θ' to θ , e.g. when revising the text.

One can impose two natural restrictions on this group [31]. These restrictions follow the general idea that the meaning of θ as probabilities of certain events is conserved during the transformation.

First, the words that have strictly zero probability $\theta_k = 0$ will stay zero, i.e. $\theta'_k = 0$ if and only if $\theta_k = 0$.

Second, the probability mixtures are conserved: if

$$\theta = \lambda \chi + (1 - \lambda) \eta, \quad 0 < \lambda < 1, \quad (55)$$

where $\chi = (\chi_1, \dots, \chi_n)$ and $\eta = (\eta_1, \dots, \eta_n)$ are arbitrary probability vectors, and where λ is a (mixing) parameter, then

$$\theta' = \lambda \chi' + (1 - \lambda) \eta'. \quad (56)$$

Here primed and non primed probability vectors relate to each other via the sought group.

The only group that (for $n \geq 3$) is consistent with the above two conditions is [31]:

$$\theta'_k = \frac{\tau_k \theta_k}{\sum_{l=1}^n \tau_l \theta_l}, \quad \tau_k > 0, \quad k = 1, \dots, n, \quad (57)$$

where τ_k are the group parameters. If the author wants to increase two times the probability of the word w_1 , then $\tau_1 = 2$ and $\tau_{k \geq 2} = 1$.

Note that (57) becomes the Bayes formula if we relate τ_k to a conditional probability [32]. In this alternative interpretation of (57), the author has to retrieve a word w having certain specific features (i.e. it is a transitive verb) from the set of words w_1, \dots, w_n having probabilities $\theta_1, \dots, \theta_n$. If we denote by $\Pr(E|w = w_k)$ the conditional probability that the word w_k displays the needed feature E , we can relate in (57) $\tau_k = \Pr(E|w = w_k)$, and (57) will describe the searching process for the word having the needed feature E .

15. Since $P(\theta)$ is the basic description of the mental lexicon that enters into our model, and once (57) is an operation by which the text is ultimately written, it is natural to suppose that the features of $P(\theta)$ can be explained by checking its response to (57). It is with a similar purpose of motivating the prior likelihood that (57) is applied in Bayesian statistics [31, 32]. There, however, the attention is focused on the non-informative prior likelihood that will stay invariant under (57). This is not suitable for our purpose precisely because we expect that the mental lexicon—whose organization $P(\theta)$ refers to—will somehow reflect the basic mechanism (57), i.e. $P(\theta)$ will display specific changes under (57).

In interpreting those changes, we adapt (57) to the probability increase of a single word w_1 , whose probability the author decides to increase by $\tau_1 > 1$ times. Thus, (57) is applied for $\tau_2 = \dots \tau_n = 1$:

$$\theta'_1 = \frac{\tau_1 \theta_1}{1 + (\tau_1 - 1) \theta_1}, \quad \theta'_l = \frac{\theta_l}{1 + (\tau_1 - 1) \theta_1}, \quad \text{for } l \geq 2. \quad (58)$$

The inverse of transformation (58) reads

$$\theta_1 = \frac{\tau_1^{-1} \theta'_1}{1 + (\tau_1^{-1} - 1) \theta'_1}, \quad \theta_l = \frac{\theta'_l}{1 + (\tau_1^{-1} - 1) \theta'_1}. \quad (59)$$

In the frequency range we are interested in, $(\tau_1^{-1} - 1) \theta'_1$ can be neglected, hence (59) just reduces to the scaling transformation:

$$\theta_1 = \tau_1^{-1} \theta'_1, \quad \theta_l = \theta'_l. \quad (60)$$

The change of the marginal likelihood for θ_1 is deduced from (53, 60):

$$P'(\theta'_1) = \frac{1}{\tau_1} P\left(\frac{\theta'_1}{\tau_1}\right) = \frac{1}{\tau_1} \left(\frac{c}{n} + \frac{\theta'_1}{\tau_1}\right)^{-2}. \quad (61)$$

Thus, for the ratio of the new to the old likelihood of the probability θ'_1 we get

$$P'(\theta'_1)/P(\theta'_1) = \tau_1 > 1 \quad \text{for } \theta'_1 \gg c\tau_1/n, \quad (62)$$

$$= \tau_1^{-1} < 1 \quad \text{for } \theta'_1 \ll c\tau_1/n. \quad (63)$$

The meaning of (62) is that once the author decides to increase the probability of the word w_1 by τ_1 times, this word will be τ_1 times more likely produced with the higher probabilities, and τ_1 times less likely with smaller probabilities; see (63). The feature is unique to the form (53) of the marginal likelihood, which by itself is due to the form (52) of $u(\theta)$. This is the mechanism that ensures the appearance of the keywords in the Zipfian range.

If $P(\theta)$ is assumed to reflect the organization of the mental lexicon, then according to (62, 63) this organization is efficient, because the decision on increasing the probability of w_1 translates to increasing the likelihood of larger values of the probability. The organization is also stable, because the likelihood at large probabilities does increase right at that amount the author planned (not more).

16. Above we related the prior likelihood $P(\theta)$ to the organization of the mental lexicon. Now we would like to clarify this relation by looking at some alternative forms of the marginal likelihood, e.g.

$$\tilde{u}(\theta) \propto (\tilde{c}n^{-1} + \theta)^{-1}, \quad (64)$$

which will produce

$$\tilde{P}(\theta) = (\tilde{c}n^{-1} + \theta)^{-1} e^{-n\theta\tilde{\mu}}. \quad (65)$$

Here $\tilde{\mu}$ is determined from

$$\int_0^\infty \frac{dy(y-1)}{\tilde{c}+y} e^{-\tilde{\mu}y} = 0, \quad (66)$$

by analogy to (12) of the main text.

It is clear that instead of (62), we now get $P'(\theta'_1)/P(\theta'_1) = 1$, i.e. the likelihood of large probabilities does not change at all. This indicates on the lack of organization in the mental lexicon (or at least very inefficient organization). The rank-frequency relation generated by (65) will read by analogy to (11) of the main text

$$\frac{r}{n} = \frac{\int_{\phi_r n}^\infty \frac{dy}{\tilde{c}+y} e^{-\tilde{\mu}y}}{\int_0^\infty \frac{dy}{\tilde{c}+y} e^{-\tilde{\mu}y}}. \quad (67)$$

In the limit of a sufficiently small \tilde{c} , the rank-frequency relation obtained from (67) is exponential,

$$\phi_r \simeq \alpha n e^{-\alpha n r}, \quad \alpha = \ln(1/\tilde{c}), \quad (68)$$

instead of the Zipf's law. According to (68) the majority of words have negligible frequencies, hence a small group

of high-frequency words dominates the text. Intuitively, this connects well with the above statement on the lack of organization.

17. The message of (62, 63) closely relates (but is not completely identical) to the word-frequency effect well-known for the mental lexicon: more frequently used words are produced (recalled) more easily [30, 34, 35]. In the context of (62, 63) this implies that the words that are decided to appear with more probability (e.g. the keywords) will be more likely produced with higher probabilities.

Note that there is no contradiction between the message of (62, 63) and the fact that all the words have the same marginal apriori likelihood [see (53)]. The latter aspect refers to the word as emerging from the mental lexicon, while the former implicitly refers to the initial stages of writing the text.

The same distinction is well known for the proper word-frequency effect in speech production, i.e. producing words from the mental lexicon [34]. According to the accepted model [34] of this process, during the first stage of speech production the author conceptualizes his thought into the abstract form of the word (lemma). This form reflects the meaning of the word and its syntactic usage, but is not yet to be put in syllabic form and pronounced [34]. The word-frequency effect comes into play during

this second stage, but is absent when the lemma is activated in the mental lexicon [34]. This is why the word-frequency can be even reversed—more frequent words are recognized more easily—for those tasks (e.g. recognition) that include mainly the lemma activation [35].

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